# Lab 05 – Z Transform and its Applications

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# Objectives

The z-transform is a useful tool in the analysis of discrete-time signals and systems and is the discrete time counterpart of the Laplace transform for continuous-time signals and systems. The z-transform may be used to solve constant coefficient difference equations, evaluate the response of a linear time-invariant system to a given input, and design linear filters. In this lab, we will look at the z-transform and examine how it may be used to solve a variety of different problems.

As compared with Fourier Transforms, there are two shortcomings to the Fourier transform approach. First, there are many useful signals in practice— such as u(n) and nu(n)—for which the discrete-time Fourier transform does not exist. Second, the transient response of a system due to initial conditions or due to changing inputs cannot be computed using the discrete-time Fourier transform approach. Therefore we now consider an extension of the discrete-time Fourier transform to address these two problems. This extension is called the z-transform. Its bilateral (or two-sided) version provides another domain in which a larger class of sequences and systems can be analyzed, and its unilateral (or one-sided) version can be used to obtain system responses with initial conditions or changing inputs.

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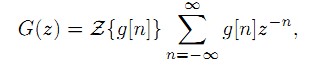
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## Background Review

However in order for any system or signal series to converge, it is necessary that the signal be absolutely summable. Unfortunately, many of the signals that we would like to consider are not absolutely summable and, therefore, do not have a DTFT. Some examples include



The z-transform is a generalization of the DTFT that allows one to deal with such sequences. The ztransform G(z) of a sequence g[n] is defined as



Where z= rejω is a complex variable.

The inverse z-transform of a complex function X(z) is given by



where C is a **counterclockwise** contour encircling the origin and lying in the ROC.

## Properties and Comments

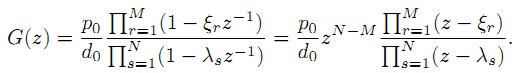
1. The complex variable z is called the complex frequency given by z = |z|ejω, where |z| is the magnitude and ω is the real frequency
2. The values of z for which the sum converges define a region in the z-plane referred to as the region of convergence (ROC). The z-transform may be viewed as the DTFT of an exponentially weighted sequence.



1. In the case of LTI discrete-time systems, all pertinent z-transforms are rational functions of z−1, that is, they are ratios of two polynomials in z−1:

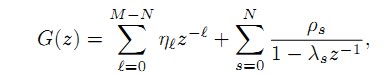


Which can be alternately written in factored form as



The roots of the numerator polynomial, ξr, are referred to as the zeros of X(z), and the roots of the denominator polynomial, λs, are referred to as the poles. There are additional N − M zeros at z=0 (the origin in the z-plane) if N>M or additional M − N poles at z=0 if N<M.

For a sequence with a rational z-transform, the ROC of the z-transform cannot contain any poles and bounded by the poles.A rational z-transform G(z)= P(z) /D(z) , where the degree of the polynomial P(z) is M and the degree of the polynomial D(z) is N, and with distinct poles at z=λs,s=1 ,2 ,...,N, can be expressed in a partial-fraction expansion form given by



1. **Convolution**



This property transforms the time-domain convolution operation into a multiplication between two functions. It is a significant property in many ways. First, if X1(z) and X2(z) are two polynomials, then their product can be implemented using the conv function in MATLAB.

### Task 1 . Solve the following convolution problem



From the definition of the z-transform, we observe that x1(n) = {2 , 3, 4} and x2(n) = {3 , 4, 5, 6} Then the convolution of these two sequences will give the coefficients of the required polynomial product.

**x1 = [2,3,4];**

**x2 = [3,4,5,6];**

**x3=conv(x1,x2);**

**x3 = [6 17 34 43 38 24]**

Ans. X3(z) = 6 + 17z−1 + 34z−2 + 43z−3 + 38z−4 + 24z−5

### Task 1b . Solve the following Convolution Problem

Let X1(z) = z +2+3z−1 and X2(z)=2z2 + 4z +3+5z−1.

1. Determine X3(z) = X1(z)X2(z)

x1=[1 2 3];

x2=[2 4 3 5];

x3=conv(x1, x2)

x3 = 2 8 17 23 19 15

1. Write down the results obtained in equation form

## B. The Zplane

ATLAB function roots on both the numerator and the denominator polynomials. (Its inverse function poly determines polynomial coefficients from its roots, as discussed in the previous section.) It is also possible to use MATLAB to plot these roots for a visual display of a pole-zero plot. The function zplane(b,a) plots poles and zeros, given the numerator row vector b and the denominator row vector a. As before, the symbol o represents a zero and the symbol x represents a pole. The plot includes the unit circle for reference. Similarly, zplane(z,p) plots the zeros in column vector z and the poles in column vector p. Note very carefully the form of the input arguments for the proper use of this function.

### Task 2 a: Given a causal system determine H(z) and sketch its pole-zero plot



The difference equation can be put in the form

y(n) − 0.9y(n − 1) = x(n)

or using equation 4.21



Use MATLAB to analyses result

**>> b = [1, 0];**

**a = [1, -0.9];**

**zplane(b,a)**

Questions

1. What does b and a represent
2. Is system casual, Explain the reason



As the pole lies within the unit circle, the system is causal.

### Task2 Analyzing Zplane using matlab

Repeat the same questions as above for the below equation



b=[0,1,1];

a=[1,-0.9,0.81];

zplane(b,a)



The system is causal because the poles are within the unit circle.

## D. The Poly Command

The function poly, which computes the polynomial coefficients, the poly function converts the roots back to polynomial coefficients. When operating on vectors, poly and roots are inverse functions, such that poly(roots(p)) returns p (up to roundoff error, ordering, and scaling). When operating on a matrix, the poly function computes the characteristic polynomial of the matrix

### Task 3 a Use poly function to find Coefficients

For the following function find the coefficients



**Matlab Code**

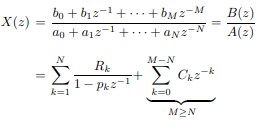
**>> b = [0,1]; = 1; a = poly([0.9,0.9,-0.9])**

**a =**

**1.0000 -0.9000 -0.8100 0.7290**

## C. Residues and Poles

A MATLAB function **residuez** is available to compute the residue part and the direct (or polynomial) terms of a rational function in z−1. Let



The **[R,p,C]=residuez(b,a)** computes the residues, poles, and direct terms of X(z) in which two polynomials B(z) and A(z) are given in two vectors b and a, respectively. The returned column vector R contains the residues, column vector p contains the pole locations, and row vector C contains the direct terms

Similarly, **[b,a]=residuez(R,p,C),** with three input arguments and two output arguments, converts the partial fraction expansion back to polynomials with coefficients in row vectors b and a

### Task 4 a Using residue command solve the rational Z transform

To check our residue calculations, let us consider the rational function



**Matlab Code**

**>> b = [0,1]; a = [3,-4,1];**

**[R,p,C] = residuez(b,a)**

R = 0.5000 -0.5000 p = 1.0000 0.3333 c = []

Or We can write the above obtained values as



Similarly, to convert back to the rational function form

**>> [b,a] = residuez(R,p,C)**

It will give us

b = 0.0000  
 0.3333

a = 1.0000   
 -1.3333   
 0.3333

Which can be written as





### Task4 b Analyzing results obtained from residue Matlab Command

Repeat the same procedure as above for the equation given in task. However replace the 3z2 with 5z3

1. What is the value of R and P

Residues of the partial fraction, specified as a vector.

R = [-0.0909 0.1877 -0.0968]

Poles of the partial fraction, specified as a vector.

p =[-1.0000 0.7236 0.2764]

C = []

1. What is C. What does it represent?

Direct terms of partial fraction are represented by C.

1. Write down the equation using value of RP?
2. Convert back to the rational function form

b = [-0.0000 0.2000 0.0000]

a = [1.0000 0.0000 -0.8000 0.2000]

## D. Inverse Z transform

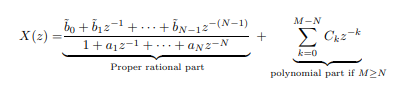
The inverse z-transform computation requires an evaluation of a complex contour integral that, in general, is a complicated procedure. The most practical approach is to use the partial fraction expansion method. The z-transform, however, must be a rational function. This requirement is generally satisfied in digital signal processing.

**Central Idea**

When X(z) is a rational function of z−1, it can be expressed as a sum of simple factors using the partial fraction expansion. The individual sequences corresponding to these factors can then be written down using the z-transform table. The inverse z-transform procedure can be summarized as follows:



Which can be expressed as



where the first term on the right-hand side is the proper rational part, and the second term is the polynomial (finite-length) part. This can be obtained by performing polynomial division if M ≥ N using the deconv function

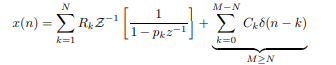
where pk is the kth pole of X(z) and Rk is the residue at pk. It is assumed that the poles are distinct for which the residues are given by



For repeated poles the expansion has a more general form. If a pole pk has multiplicity r, then its expansion is given by



where the residues Rkl are computed using a more general formula, which is available in reference [23]. Assuming distinct poles we can write x(n) as



### Task 5a Compute Z transform using residuez Command

Compute inverse z transform



By analyzing the above equation we get,

b=[1]

a= [1.0000 -0.9000 -0.8100 0.7290]

below command is to be used

[R,p,C]=residuez(b,a)

### 5c Verifying inverse Z transform using matlab

Determine the inverse z-transform of



**b = [1,0.4\*sqrt(2)]; a=[1,-0.8\*sqrt(2),0.64]; ‘**

**[R,p,C] = residuez(b,a)**

**Mp=(abs(p))’ % pole magnitudes**

**Ap=(angle(p))’/pi % pole angles in pi units**

**Questions**

1. What is R P C

R =[0.5000 - 1.0000i 0.5000 + 1.0000i]

p = [0.5657+0.5657i 0.5657-0.5657i]

1. What is Mp and Mp

Mp = [0.8000 0.8000]

Ap =[0.2500 -0.2500]

1. Can you corelate your results with following equation , and explain the process



1. Using Table 4.1 . an you covert above equation to



1. Explain the process
2. Does the equation given in problem3c and the final equation represent same sequence.
3. You mar check he result using following code. Note don your observation

**[delta, n] = impseq(0,0,6)**

**x = filter(b,a,delta) % check sequence**

**>> x = ((0.8).^n).\*(cos(pi\*n/4)+2\*sin(pi\*n/4))**

**x =**

**1.0000 1.6971 1.2800 0.3620 -0.4096 -0.6951 -0.5243**

**x =**

**1.0000 1.6971 1.2800 0.3620 -0.4096 -0.6951 -0.5243**

**Sequence is same.**

## E. freqz Commnad

The Command **[H,w] = freqz(b,a,N)** returns the N-point frequency vector w and the N-point complex frequency response vector H of the system, given its numerator and denominator coefficients in vectors b and a. The frequency response is evaluated at N points equally spaced around the upper half of the unit circle. Note that the b and a vectors are the same vectors we use in the filter function or derived from the difference equation representation. The second form [**H,w] = freqz(b,a,N,’whole’)** uses N points around the whole unit circle for computation. In yet another form **H = freqz(b,a,w)** it returns the frequency response at frequencies designated in vector w, normally between 0 and π. It should be noted that the freqz function can also be used for numerical computation of the DTFT of a finite-duration, causal sequence x(n). In this approach, b=x and a=1

### Task 6 a. Determining |H(ejω)| and H(ejω).

For the equation given in task 2-a,

a. Determine H(z) and sketch its pole-zero plot.

b. Plot magnitude and phase of |H(ejω)|

**Hints to Solve Solution**

Step1 : Find Zplane – Discuss The result

Diagram

Description automatically generated

Step 2 : Use MATLAB to illustrate the use of the freqz function and take 100 points

Step 3 You can use the following command [H,w] = freqz(b,a,100)

Step 4 Plot and calculate magnitude and phase



# Post Lab

A causal, linear, and time-invariant system is given by the following difference equation:



Find the system function H (z) for this system.

b = [0 1]; %zeros

a = [1 -1 -1]; %poles

Plot the poles and zeros of H (z) and indicate the region of convergence (ROC).



A picture containing text

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The first two values are of R and the last two are P values.

Poles : P1 = -0.6180 and P2 = 1.6180

Convergence |z| > P2